

Computer-Aided Noise Analysis of Linear Multiport Networks of Arbitrary Topology

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Abstract—The paper discusses a new algorithm for the noise analysis of a linear multiport network. The circuit may include any kind of passive components introducing thermal noise only, and any number of two-port devices described by the usual four noise parameters. On output, the algorithm produces the correlation matrix of the Norton equivalent noise current sources at the network ports. The approach is suitable for implementation into any general-purpose microwave circuit design program.

I. INTRODUCTION

IN THIS PAPER, the problem of evaluating the noise properties of a general linear multiport network is tackled from a computer-aided design viewpoint. This means that the formulation of the problem is typical of a general-purpose CAD environment, i.e., the network may be defined component-wise in an arbitrary way, and may include any kind of lossy passive (possibly nonreciprocal) components, as well as any number of active two-port devices. Passive components are described by geometrical and technological data, and are assumed to introduce thermal noise only. For each active device, the starting-point information is represented by the usual four spot noise parameters [1] and by the admittance (or any equivalent) matrix; such data is assumed to be available from measurements and/or physical circuit models. The purpose of our work is to introduce a computer algorithm allowing a straightforward calculation of the noise properties of the overall network, regardless of its topology.

In the general multiport case, a possible approach (actually the one to be developed here) is to represent the noisy network by its Norton equivalent, that is, by its noiseless counterpart with a noise current source connected across each port. This situation is schematically represented in Fig. 1. Quite obviously, such sources are not statistically independent of each other, so that a complete analysis must include the computation of their correlation matrix. As a typical application, this information may be required to determine the noise behavior of a nonlinear circuit embedding the linear multiport under consideration, such as a microwave front-end including a multiple-diode mixer [2]. In the special but important two-port case, the usual

description in terms of spot noise parameters can be obtained from the correlation matrix by means of explicit algebraic expressions. This ultimately leads to the knowledge of the noise figure once the source admittance has been specified.

The noise analysis of microwave circuits of complex topology has received some attention in the technical literature. Formulas for evaluating the noise parameters of a combination of two-ports (one of which noiseless) have been available for a while (e.g., [3]). More recently, expressions for the noise figure of some popular configurations, such as the feedback and the distributed amplifier, have been presented [4]–[6]. These approaches are far from providing a complete solution to the CAD problem because of their limiting assumptions and/or their specialized computational schemes that are only applicable to preselected topologies. On the contrary, the method proposed in this paper is completely general and can readily be implemented into any general-purpose circuit design program.

II. BASIC RELATIONSHIPS FOR A NOISY TWO-PORT

The classic equivalent representation of a linear noisy two-port [1] is given in Fig. 2(a). For spot noise calculations, E and J are complex phasors and have the meaning of spectral (pseudo-sinusoidal) components of the corresponding noise voltage and current at a given frequency f .

When the two-port is driven by a sinusoidal source of internal admittance $Y_S = G_S + jB_S$, its noise figure has the well-known expression

$$F = F_{\min} + \frac{R_N}{G_S} |Y_S - Y_0|^2 \quad (1)$$

where F_{\min} is the minimum noise figure, $Y_0 = G_0 + jB_0$ is the optimum source admittance, R_N is the equivalent noise resistance of the voltage source E . F_{\min} , G_0 , B_0 , and R_N are the four spot noise parameters of the linear two-port at the frequency of interest.

In general, E and J are not statistically independent; if we denote by J_{cor} that component of J which is completely correlated with E , we may let

$$J_{\text{cor}} = Y_{\text{cor}} E \quad (2)$$

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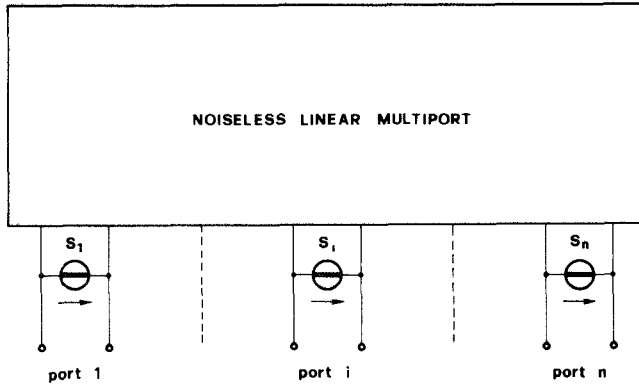


Fig. 1. Norton's equivalent circuit of a noisy linear multiport.

defining the correlation admittance $Y_{\text{cor}} = G_{\text{cor}} + jB_{\text{cor}}$. From [1], we obtain

$$\begin{aligned} G_{\text{cor}} &= \frac{F_{\text{min}} - 1}{2R_N} - G_0 \\ B_{\text{cor}} &= -B_0 \\ G_N &= R_N(G_0^2 - G_{\text{cor}}^2) \end{aligned} \quad (3)$$

where G_N is the equivalent noise conductance of $J - J_{\text{cor}}$. Thus, by definition

$$\begin{aligned} \langle |E|^2 \rangle &= 4K_B T_0 R_N \Delta f \\ \langle |J|^2 \rangle &= 4K_B T_0 (|Y_{\text{cor}}|^2 R_N + G_N) \Delta f \\ \langle EJ^* \rangle &= Y_{\text{cor}}^* \langle |E|^2 \rangle \end{aligned} \quad (4)$$

where

- K_B Boltzmann's constant,
- T_0 reference absolute temperature,
- Δf noise bandwidth,
- $\langle \rangle$ statistical (ensemble) average,
- $*$ complex conjugate.

For the present purposes, it is more convenient to make use of the Norton equivalent circuit of the noisy two-port shown in Fig. 2(b). If we denote by y_{pq} ($p, q = 1, 2$), the elements of the admittance matrix at frequency f , by inspection of Fig. 1, we get

$$\begin{aligned} J_1 &= J - y_{11}E \\ J_2 &= -y_{21}E. \end{aligned} \quad (5)$$

The correlation matrix of the noise current sources J_1, J_2 is defined as

$$\mathcal{C}_J = [\langle J_p J_q^* \rangle] = 4K_B T_0 \Delta f C_J \quad (p, q = 1, 2). \quad (6)$$

Combining (5) with (4) yields the normalized correlation matrix C_J as a function of the admittance and noise parameters

$$C_J = \begin{bmatrix} G_N + |y_{11} - Y_{\text{cor}}|^2 R_N & y_{21}^* (y_{11} - Y_{\text{cor}}) R_N \\ y_{21} (y_{11} - Y_{\text{cor}})^* R_N & |y_{21}|^2 R_N \end{bmatrix}. \quad (7)$$

Conversely, the correlation matrix can be used to derive the four noise parameters. A straightforward manipulation

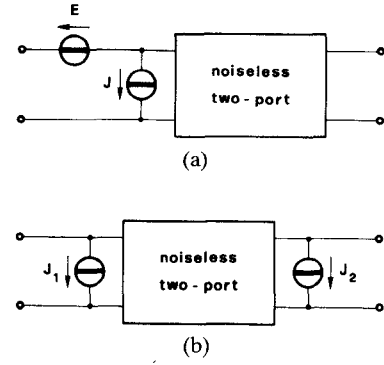


Fig. 2. Classic equivalent representations of a linear noisy two-port.

of (7) yields

$$\begin{aligned} R_N &= \frac{1}{|y_{21}|^2} \cdot \frac{\langle |J_2|^2 \rangle}{4K_B T_0 \Delta f} \\ Y_{\text{cor}} &= G_{\text{cor}} + jB_{\text{cor}} = y_{11} - y_{21} \frac{\langle J_1 J_2^* \rangle}{\langle |J_2|^2 \rangle} \\ G_N &= \frac{\langle |J_1|^2 \rangle}{4K_B T_0 \Delta f} - \frac{\langle J_1 J_2^* \rangle}{\langle |J_2|^2 \rangle} \cdot \frac{\langle J_1^* J_2 \rangle}{4K_B T_0 \Delta f}. \end{aligned} \quad (8)$$

Then from (3)

$$\begin{aligned} B_0 &= -B_{\text{cor}} \\ G_0 &= \sqrt{\frac{G_N + R_N G_{\text{cor}}^2}{R_N}} \\ F_{\text{min}} &= 1 + 2R_N(G_0 + G_{\text{cor}}). \end{aligned} \quad (9)$$

Note that, from (1) by means of (9) and (8) after some algebraic manipulations, we obtain

$$\begin{aligned} 4K_B T_0 G_S \Delta f (F - 1) &= \langle |J_1|^2 \rangle + \left| \frac{y_{11} + Y_S}{y_{21}} \right|^2 \langle |J_2|^2 \rangle \\ &\quad - 2 \operatorname{Re} \left[\frac{y_{11} + Y_S}{y_{21}} \langle J_1^* J_2 \rangle \right]. \end{aligned} \quad (10)$$

According to (10), given a source admittance Y_S , $F - 1$ can be expressed as a homogeneous linear function of the elements of the correlation matrix (6), as it was easily predictable from a physical viewpoint. Equation (10) can also be obtained directly from the equivalent circuit of Fig. 2(b) by applying the definition of noise figure.

III. ANALYSIS OF A GENERAL NOISY MULTIPOINT

We consider a linear noisy n -port consisting of a lossy passive network embedding a number (namely m) of noisy two-port devices. Each device is assumed to be characterized by its admittance matrix and its four spot noise parameters at each frequency of interest. As shown in Fig. 3, the given network may be represented as the interconnection of a passive $(2m + n)$ -port with the m two-port devices. In the following, the $(2m + n)$ -port will be referred to as the passive network. This network may result from an

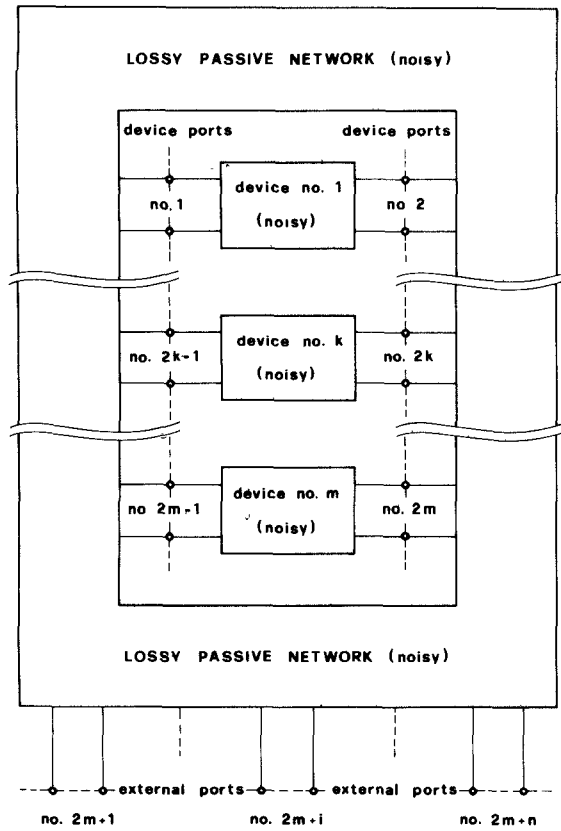


Fig. 3. General decomposition of a network embedding a number of noisy two-port devices.

arbitrary interconnection of the usual passive microwave components and may be nonreciprocal, provided that its only noise contribution be represented by the thermal noise sources associated with ohmic losses.

To derive the equivalent circuit of Fig. 1, we first replace each noisy network appearing in Fig. 3 by its Norton counterpart, thus generating the configuration shown in Fig. 4.

In this figure, two independent sets of noise current sources show up. The N -sources are the equivalent noise generators of the passive network and are thermal in nature; they are not statistically independent, and their correlation matrix has the expression [7]

$$\begin{aligned} \mathcal{C}_N &= [\langle N_p N_q^* \rangle] \\ &= 2K_B T_0 \Delta f (Y + Y^*) \quad (p, q = 1, 2, \dots, 2m + n) \end{aligned} \quad (11)$$

where Y is the $(2m + n) \times (2m + n)$ admittance matrix of the passive network, and $*$ applied to a complex matrix indicates the conjugate transposed.

The J -sources are the equivalent noise generators of the two-port devices and are usually not only thermal. Obviously, only the couples of sources associated with the same device, namely J_{2k-1}, J_{2k} for the k th, are correlated, and their normalized correlation matrix C_{Jk} has the expression (7).

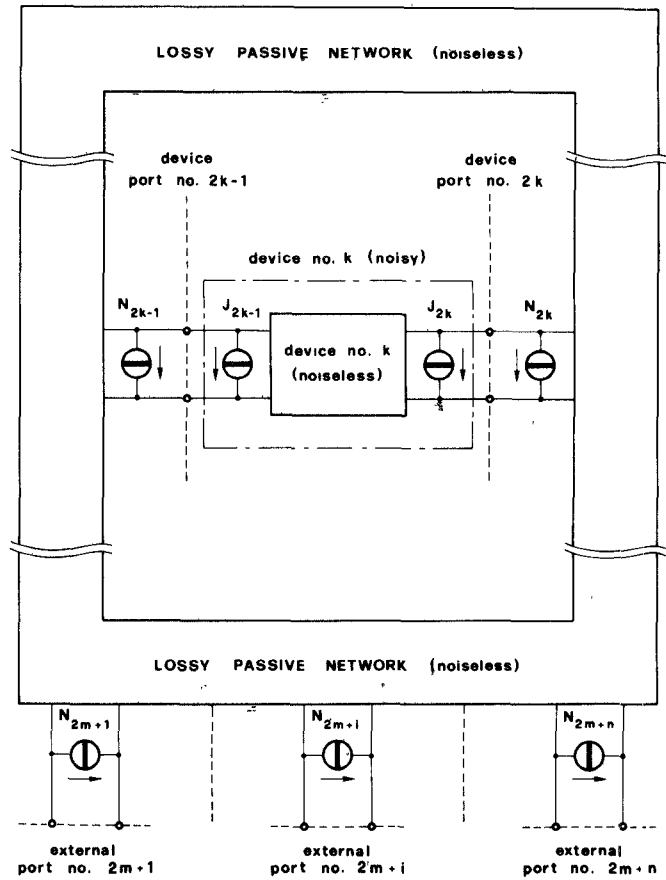


Fig. 4. Norton's equivalent representation of the network depicted in Fig. 3.

Now let the admittance matrix be partitioned as follows:

$$Y = \begin{bmatrix} Y_{dd} & Y_{de} \\ Y_{ed} & Y_{ee} \end{bmatrix} \quad (12)$$

where the subscript d refers to the $2m$ device ports, and the subscript e to the n external ports (Fig. 3). Accordingly, the network equations take the form (Fig. 4).

$$\begin{cases} I_d = Y_{dd} V_d + Y_{de} V_e + N_d \\ I_e = Y_{ed} V_d + Y_{ee} V_e + N_e \\ I_d = -y V_d - J \end{cases} \quad (13)$$

where y is the diagonal sum of the device admittance matrices and the vectors of noise current phasors have been partitioned in a way similar to (12).

The equivalent circuit of Fig. 1 may be derived by short circuiting the external ports ($V_e = 0$), in which case $I_e = S$ (Fig. 1). Then from (13)

$$\begin{cases} -y V_d - J = Y_{dd} V_d + N_d \\ S = Y_{ed} V_d + N_e \end{cases} \quad (14)$$

The solution of (14) can be written in the form

$$S = H_N N + H_J J \quad (15)$$

where

$$\begin{aligned} H_J &= -Y_{ed} (Y_{dd} + y)^{-1} \\ H_N &= [H_J \mid I_n] \end{aligned} \quad (16)$$

and

$$N = \begin{bmatrix} N_d \\ \overline{N_e} \end{bmatrix}. \quad (17)$$

In (16), we denote by I_n the identity matrix of order n .

Since the N and J sources are statistically independent, the effects of the two terms on the right-hand side of (15) may be superimposed in power. Thus, the correlation matrix is given by

$$\langle SS^* \rangle = H_N \langle NN^* \rangle H_N^* + H_J \langle JJ^* \rangle H_J^* \quad (18)$$

or, from (11) and (6)

$$\langle SS^* \rangle = 2K_B T_0 \Delta f [H_N (Y + Y^*) H_N^* + 2H_J C_J H_J^*] \quad (19)$$

where C_J is the diagonal sum of the normalized correlation matrices C_{Jk} of the two-port devices.

Note that (19) provides separate explicit expressions for the two contributions to the correlation matrix, that is, the thermal noise of the passive network and the noise injected by the active devices. In the special case of a two-port ($n = 2$), the same conclusion automatically applies to the noise figure thanks to (10), once a source admittance has been specified. Furthermore, in the two-port case, (8) and (9) can be used to derive from (19) the four spot noise parameters in a straightforward way.

As a final point, we observe that the admittance matrix of the overall n -port network of Fig. 1 may be written in the form

$$Y_L = Y_{ee} + H_J Y_{de} \quad (20)$$

and can be obtained with a negligible increase of computational effort once Y and H_J have been found. Thus, a conventional analysis of the circuit may be carried out as a by-product of the noise calculations. This allows a network of absolutely general topology to be simultaneously optimized with respect to both the noise properties and any of the conventional network functions.

IV. APPLICATIONS

The algorithm described in the previous sections was implemented into a general-purpose microwave CAD program based on the subnetwork-growth method (SGM) of circuit analysis [8]. When the noise analysis is requested by the user at the data entry level, a preliminary section of the program, running only once for a given analysis or design, generates the passive network of Fig. 3 by elimination of the devices from the original user-defined topology. The passive network is then analyzed in the usual way, and a call to a special subroutine performing the steps of the algorithm is generated. Thus, the results of the noise analysis become available to the user together with conventional electrical information (e.g., the scattering matrix) in a completely transparent way, no matter what the circuit topology. When a noise analysis is not required, the special sections of the program are bypassed, and only the standard analysis is carried out. An important point is that the same subroutines can be used in both cases to perform the

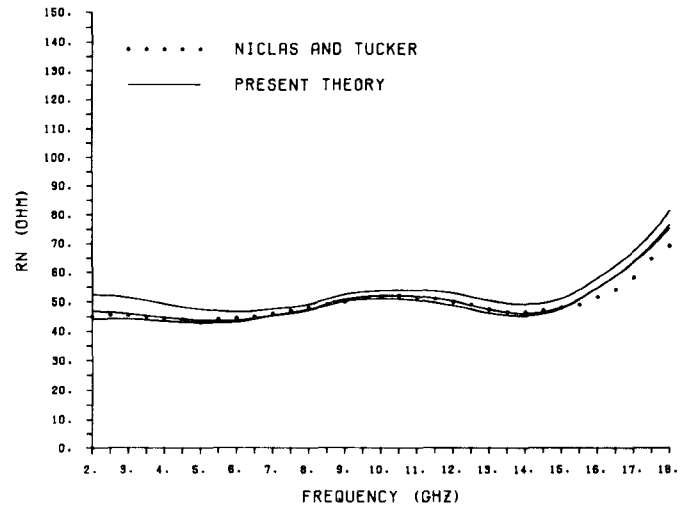


Fig. 5. Equivalent noise resistance of a distributed amplifier [4]. The three solid curves are associated with different values of R_S .

interconnections of the circuit components, so that only minor changes have to be made in an existing program in order to implement the noise calculations.

When using the SGM, the CPU time required for a full analysis (including noise) is larger than for a conventional one, due to the nonoptimum sequence of interconnections, leading to the inversion of a relatively large-order matrix (16). The time penalty is directly dependent on the number of noisy devices contained in the circuit. As an example, for a two-port network including three devices (which means an eight-port passive network) this penalty is about 85 percent.

As a typical application, we carried out a full noise analysis of the three-stage distributed amplifier described by Niclas *et al.* [5], [9]. Based on the data reported in [5] and [9], the noise correlation matrix was computed from (19) throughout the band of interest. The four spot noise parameters and the noise figure for a 50- Ω source impedance were then derived by means of (8), (9), and (10). Making use of the first term in square brackets on the right-hand side of (19) and again of (10), we also computed the contribution F_{id} to the amplifier noise figure due to the passive network alone (i.e., in the case of noiseless FET's). All calculations were performed in the cases of lossless microstrip lines (as in [5]) and for two different values of the microstrip skin resistance at 10 GHz, namely $R_S = 0.1 \Omega$ and $R_S = 0.2 \Omega$. Microstrip losses were evaluated by the formulas given in [10].

The numerical results are presented in Figs. 5–9, where a comparison with the calculations by Niclas and Tucker [5] is also provided. For $R_S = 0$, the two sets of curves closely match, with minor discrepancies that can probably be ascribed to the use of slightly different microstrip models. The remaining curves show the effects of microstrip losses on the noise performance of the amplifier. Such effects are most significant on the minimum noise figure F_{min} ; the remaining noise parameters are almost left unchanged.

Note that a typical value of R_S at 10 GHz would be around 0.05 Ω , corresponding to an attenuation constant

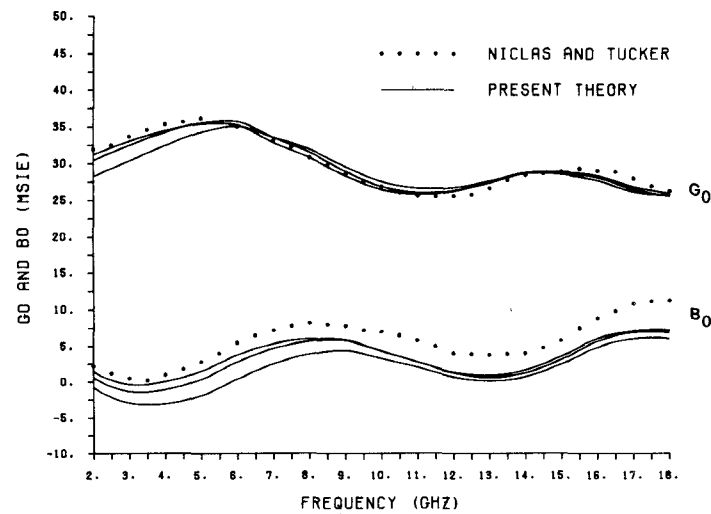


Fig. 6. Optimum noise source admittance of a distributed amplifier [4]. The three solid curves are associated with different values of R_S .

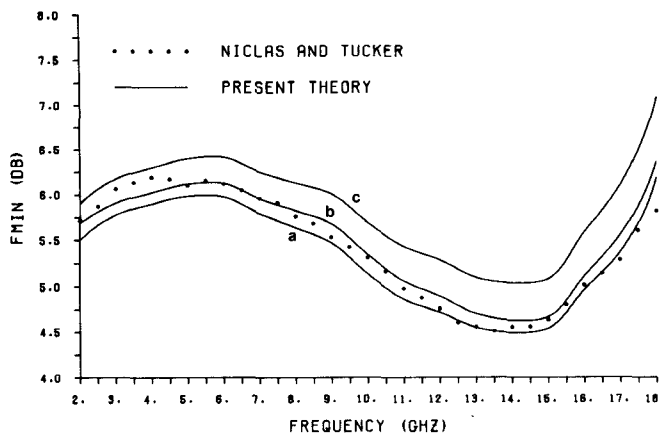


Fig. 7. Minimum noise figure of a distributed amplifier [4]. (a) $R_S = 0$ (b) $R_S = 0.1 \Omega$. (c) $R_S = 0.2 \Omega$.

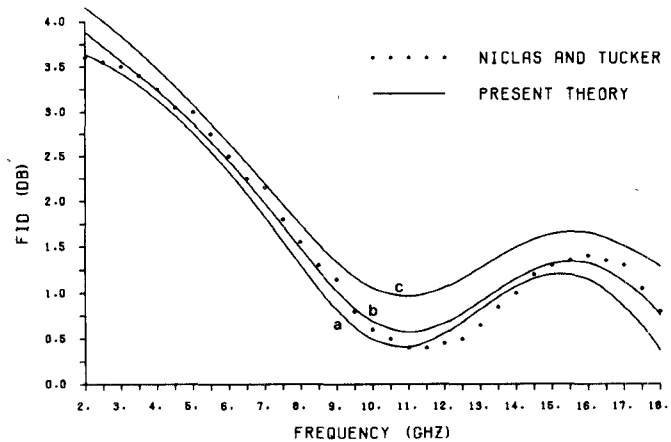


Fig. 9. Noise figure of a distributed amplifier [4] with noiseless FET's. (a) $R_S = 0$. (b) $R_S = 0.1 \Omega$. (c) $R_S = 0.2 \Omega$.

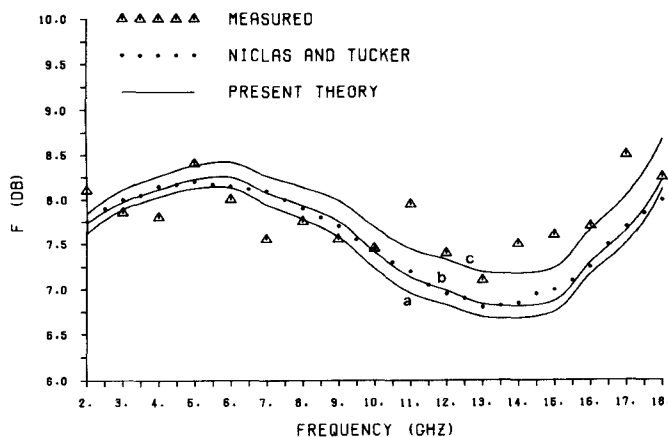


Fig. 8. Noise figure of a distributed amplifier [4]. (a) $R_S = 0$. (b) $R_S = 0.1 \Omega$. (c) $R_S = 0.2 \Omega$.

of about 0.1 dB/cm for the 50- Ω line on the quartz substrate used by Niclas *et al.* [9]. Relatively large values of R_S were used here to emphasize the applicability of our algorithm to the lossy as well as to the loss-free microstrip cases. In view of the above, the results reported in Figs.

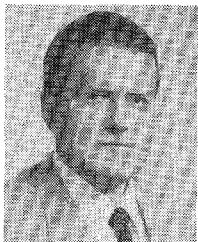
5–10 clearly support the statement [5] that microstrip losses give a negligible contribution to the amplifier noise figure in the present case. This might not be true, however, for a monolithic GaAs amplifier built (for example) on a 150- μm substrate, for which $R_S = 0.05$ would lead to an attenuation constant of about 0.39 dB/cm for the 50- Ω microstrip at 10 GHz (according to [10]).

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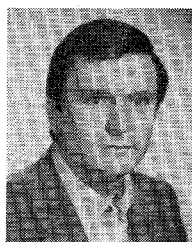


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